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Global Receiver Operating Characteristic (ROC) Curves for Three Network Topologies

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13. ABSTRACT (Maximum 200 words)

In warfare analysis, obtaining a single descriptor of a system is critical. As the focus on networking and information technology in warfare analysis intensifies, the ability to describe a system quickly and easily will be imperative. This document surveys multisensor detection work with a focus on how to combine multiple receiver operating characteristic (ROC) curves into a single network or global metric. A network ROC curve is a rapid and simple way to explain the performance of a system. While many of the formulas presented here are difficult to solve, through parametric analysis they can provide insight into trends in various warfare areas where ROC curves arise, including detection/classification, command and control, and decision-making.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

GLOBAL RECEIVER OPERATING CHARACTERISTIC (ROC) CURVES FOR THREE NETWORK TOPOLOGIES

1. INTRODUCTION

In traditional multisensor detection, local sensors transmit data to a central processor that performs optimal detection based on conventional statistical techniques. In decentralized or distributed detection, each geographically dispersed sensor executes limited data processing before transmitting condensed information to a central site. Thus, the sensor becomes a processor. Interest in distributed detection has intensified due to the relatively low cost of sensors, the inherent network redundancy, the availability of high-speed communication networks, and the increased computational capacity of sensors.[1,2]

One element of distributed detection is receiver operating characteristic (ROC) curves, which show the relationship between the probability of detection and the probability of a false alarm. This document surveys the results of multisensor detection work, with a focus on how to combine multiple ROC curves into a single network or global ROC curve to describe the network. The solution depends on the network topology. While other topologies exist, this document examines only three networks:

- a fusion network (e.g., common undersea picture),
- a serial network (e.g., large numbers of distributed sensors), and
- a tree network (e.g., command and control).

The fusion and serial networks are special cases of the tree network. For each of the three topologies considered, formulas are presented to compute the global ROC curve. The focus is on the fusion network, however, with minimal results presented for the serial and tree networks.

For each topology, the network has N local processors conducting binary hypothesis testing. Processor i receives input, decides between two hypotheses (H_0 and H_1), and outputs a decision u_i, which is determined as follows:

$$u_i = \begin{cases} 0 & \text{processor } i \text{ decides } H_0 \\ 1 & \text{processor } i \text{ decides } H_1 \end{cases} . \tag{1}$$

The processors function together to generate the best network decision D. Thus, the network decision follows an optimality criterion. The Neyman-Pearson optimality criterion, used in this analysis, constrains the global probability of false alarm $P_F \leq \delta$ and maximizes the global probability of detection P_D using Lagrangian multipliers. The Lagrangian function is $\beta(P_F - \delta) - P_D$, where β is the Lagrangian multiplier or the global threshold that yields the maximum network probability of false alarm such that $P_F \leq \delta$. The Bayesian minimum cost criterion is addressed in section 5, "Bayesian Formulation."

For this discussion, the decision-making processes of the local processors are assumed to be independent. Associated with each of the N local processors is a ROC curve. For each processor i, the probability of false alarm and the probability of detection (independent of network topology) are defined as[3]

$$P_{f_i} = \int_{\alpha_i}^{\infty} P(x \mid H_0)dx = Pr(u_i = 1 \mid H_0) \ , \tag{2a}$$

$$P_{d_i} = \int_{\alpha_i}^{\infty} P(x \mid H_1)dx = Pr(u_i = 1 \mid H_1) \ , \tag{2b}$$

where α_i is the detection threshold, $Pr(u_i = 1 \mid H_0)$ is the probability of deciding H_1 is true when H_0 is true, and $Pr(u_i = 1 \mid H_1)$ is the probability of deciding H_1 is true when H_1 is true. (Throughout this document, a lowercase subscript refers to an individual sensor, while an uppercase subscript refers to the network.) The definition of the detection threshold will depend on the problem being analyzed. For example, for a sonar detection-related problem, the detection threshold might correspond to the signal-to-noise ratio required for detection.

An issue arises at the fusion center when multiple processors decide on the same hypothesis when the problem assumptions (known at the fusion center) prevent this hypothesis from being correct. For example, a single red submarine is in a known area (area search), and N individual processors are tasked to detect the submarine in non-overlapping sub-regions over some period. How the individual decisions are combined into one solution resolves the issue of the network decision when multiple processors detect a submarine at the same time.

2. FUSION NETWORK

2.1 FUSION CENTER DOES NOT RECEIVE EXTERNAL OBSERVATION

Figure 1 shows the topology of the N-processor fusion network where the fusion center receives only the output of the individual local processors. Each local processor receives an external observation vector X_i, and outputs a decision u_i. The fusion center receives the N decisions $u_1,\ldots,u_N$, combines the individual decisions according to some fusion rule, and outputs a final network decision D.

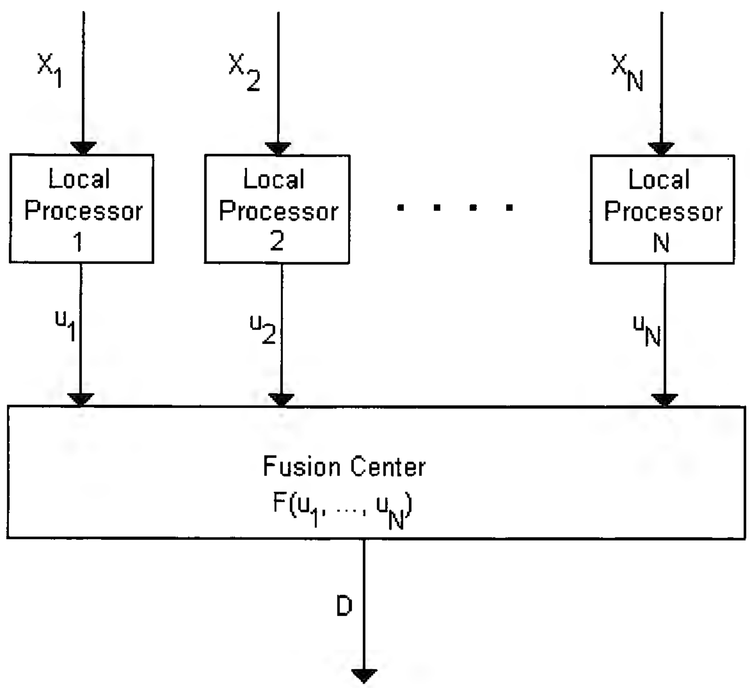

Figure 1. N-Processor Fusion Network

The general fusion rule is given by[4]

$$F(u_1,\ldots,u_N)=\begin{cases}1 & \text{if } \dfrac{P(u_1,u_2,\ldots,u_N \mid H_1)}{P(u_1,u_2,\ldots,u_N \mid H_0)} \geq \beta \\[2ex] 0 & \text{if } \dfrac{P(u_1,u_2,\ldots,u_N \mid H_1)}{P(u_1,u_2,\ldots,u_N \mid H_0)} < \beta\end{cases}, \tag{3}$$

where β is the global threshold, which is the Lagrangian multiplier for the Neyman-Pearson criterion.[4] Since the local decisions are independent, the likelihood ratio can be simplified to[4]

$$\frac{P(u_1,u_2,\ldots,u_N \mid H_1)}{P(u_1,u_2,\ldots,u_N \mid H_0)} = \prod_{i=1}^{n} \frac{P(u_i \mid H_1)}{P(u_i \mid H_0)}$$

$$= \prod_{U(1)} \frac{P(u_i = 1 \mid H_1)}{P(u_i = 1 \mid H_0)} \prod_{U(0)} \frac{P(u_i = 0 \mid H_1)}{P(u_i = 0 \mid H_0)}$$

$$= \prod_{U(1)} \frac{P_{d_i}}{P_{f_i}} \prod_{U(0)} \frac{1-P_{d_i}}{1-P_{f_i}} \qquad (4)$$

where $U(1)$ is the set of all i such that $u_i = 1$, $U(0)$ is the set of all i such that $u_i = 0$, and P_{d_i} and P_{f_i} represent the probability of detection and the probability of false alarm for processor i, respectively.

The (P_F, P_D) operating point of the fusion center is[4]

$$P_F = \sum_{u_1}\sum_{u_2}\cdots\sum_{u_N} F(u_1,\ldots,u_N) P(u_1,\ldots,u_N \mid H_0)$$

$$= \sum_{u_1}\sum_{u_2}\cdots\sum_{u_N} F(u_1,\ldots,u_N) \prod_{i=1}^{N} P(u_i \mid H_0)$$

$$= \sum_{u_1}\sum_{u_2}\cdots\sum_{u_N} F(u_1,\ldots,u_N) \prod_{U(1)} P_{f_i} \prod_{U(0)} (1-P_{f_i}), \qquad (5a)$$

and

$$P_D = \sum_{u_1}\sum_{u_2}\cdots\sum_{u_N} F(u_1,\ldots,u_N) P(u_1,\ldots,u_N \mid H_1)$$

$$= \sum_{u_1}\sum_{u_2}\cdots\sum_{u_N} F(u_1,\ldots,u_N) \prod_{i=1}^{N} P(u_i \mid H_1)$$

$$= \sum_{u_1}\sum_{u_2}\cdots\sum_{u_N} F(u_1,\ldots,u_N) \prod_{U(1)} P_{d_i} \prod_{U(0)} (1-P_{d_i}). \qquad (5b)$$

Equations (5a) and (5b) can be difficult to compute for large N.

There is one caveat when using Lagrangian multipliers: the individual ROC curves must be convex. Otherwise, the solution might not be locally optimal.

A special case that can be efficiently evaluated is when the fusion rule is a k-of-out-N logical decision (i.e., k processors decide hypothesis H_1). If the operating points of the local processors are identical, then given a point (P_f, P_d) on the individual ROC curve, equations (5a) and (5b) determine a point on the global ROC curve as[4]

$$P_F = 1 - \sum_{j=0}^{k-1} \binom{N}{j} (P_f)^j (1 - P_f)^{N-j}, \tag{6a}$$

and

$$P_D = 1 - \sum_{j=0}^{k-1} \binom{N}{j} (P_d)^j (1 - P_d)^{N-j}. \tag{6b}$$

The global performance can also be expressed in terms of the incomplete beta function[5]

$$I_x(a,b) = \frac{1}{B(a,b)} \int_0^x t^{a-1} (1-t)^{b-1} dt$$
$$= \frac{\int_0^x t^{a-1}(1-t)^{b-1} dt}{\int_0^1 t^{a-1}(1-t)^{b-1} dt}, \tag{7}$$

where $B(a,b)$ is the complete beta function. Thus, the global performance is[4]

$$P_F = I_{P_f}(k, N-k+1) \tag{8a}$$

and

$$P_D = I_{P_d}(k, N-k+1). \tag{8b}$$

If $P_f \leq 0.5$ (which is almost certainly true in applications), then $N \geq 2$. Using equations (8a) and (8b), the global probability of false alarm and global probability of detection can be expressed in terms of the approximation shown in appendix A and the optimal fusion rule k[4,5]

$$P_F(k) = Q\left(\frac{(1-P_f)^{1/3}(9k-1)(N-k+1)^{2/3} - P_f^{1/3}(9(N-k+1)-1)k^{2/3}}{3k^{1/2}\sqrt{N-k+1}\sqrt{P_f^{2/3}k^{1/3} + (1-P_f)^{2/3}(N-k+1)^{1/3}}} \right), \tag{9a}$$

and

$$P_D(k) = Q\left(\frac{(1-P_d)^{1/3}(9k-1)(N-k+1)^{2/3} - P_d^{1/3}(9(N-k+1)-1)k^{2/3}}{3k^{1/2}\sqrt{N-k+1}\sqrt{P_d^{2/3}k^{1/3} + (1-P_d)^{2/3}(N-k+1)^{1/3}}} \right). \tag{9b}$$

Equations (9a) and (9b) will produce the optimal solution if k and α are the optimal solutions to the global threshold equations.

2.2 FAULT-TOLERANT REDUNDANT NETWORK

Figure 2 shows a fault-tolerant redundant fusion network. Each of the N identical local processors receives the same observation vector X, and outputs a decision u_i. If no faults occur, the processors output identical decisions. The fusion center combines the N decisions $u_1, \ldots u_N$ to reduce the effect of a fault in any local processor, and the fusion center outputs a final decision D.

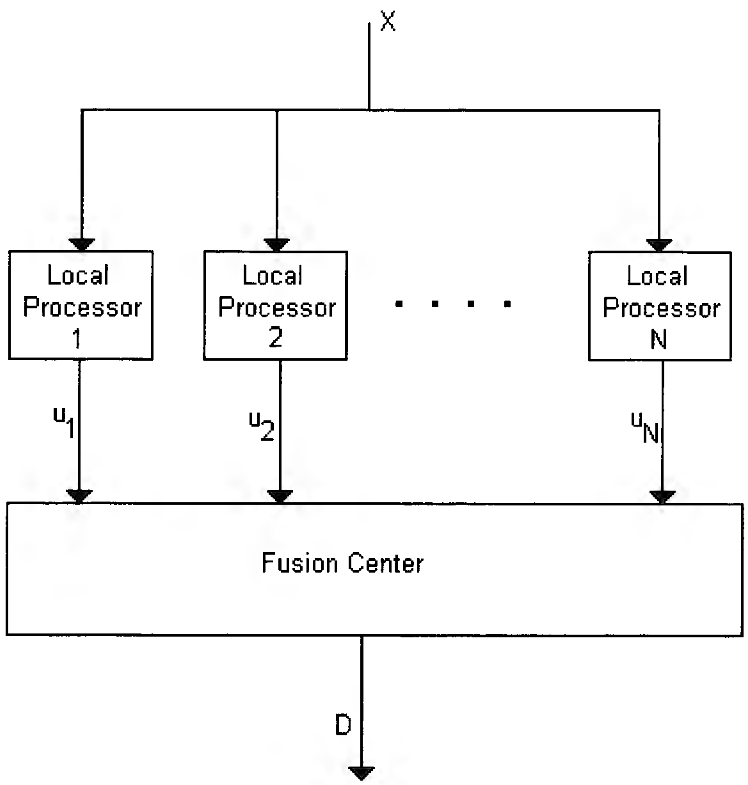

Figure 2. N-Processor Redundant Fusion Network

Network faults can be classified into one of two types: hard failures, in which the network, or one of its elements, is not able to deliver any traffic at all; and soft failures, which includes network/service anomalies or degradations in various performance parameters. (The cause of the fault is not significant, and therefore is not discussed here.) A failure occurs when the processor does not deliver the theoretically expected decision. Since the fusion processor is simpler than the local processors, the probability of failure in the fusion processor is insignificant compared to the probability of failure in the local processor. Therefore, the only faults considered in this paper are those located at the individual processors, and not at the fusion center.

Processor performance is expressed as a function of the fault-free performance (i.e., the theoretically optimal value). Assume that these probabilities are independent of the hypothesis and observation. For processor i, the probability of false alarm is[4,6]

$$\begin{aligned} P_{f_i} &= P(u_i = 1 \mid H_0) \\ &= P(u_i = 1 \mid \widetilde{u}_i = 1) P(\widetilde{u}_i = 1 \mid H_0) + P(u_i = 1 \mid \widetilde{u}_i = 0) P(\widetilde{u}_i = 0 \mid H_0) \\ &= (1 - p_0)\widetilde{P}_{f_i} + p_1\left(1 - \widetilde{P}_{f_i}\right) \\ &= p_1 + (1 - p_1 - p_0)\widetilde{P}_{f_i} , \end{aligned} \tag{10}$$

where

$u_i =$ the observed decision,
$\widetilde{u}_i =$ the theoretically optimum decision,
the superscript $\sim$ indicates that the value is the theoretically optimal value,
$P(u_i = 1 \mid u_i^* = 0) = p_1$, and

$P(u_i = 0 \mid u_i^* = 1) = p_0$.

Similarly, the probability of detection is[4,6]

$$P_{d_i} = p_1 + (1 - p_1 - p_0)\widetilde{P}_{d_i} . \tag{11}$$

Note that the original ROC curve for processor i is scaled by a factor $1 - p_1 - p_0$ in each dimension.

When the fusion rule is a k-out-of-N logical function, the probability of false alarm for the redundant network can be written as[4]

$$\begin{aligned} P_F &= P(D = 1 \mid H_0) \\ &= P(\widetilde{D} = 1 \mid H_0)\left(1 - \sum_{j=0}^{k-1}\binom{N}{j} P\left(u_i = 1 \mid \widetilde{D} = 1, H_0\right)^j P\left(u_i = 0 \mid \widetilde{D} = 1, H_0\right)^{N-j}\right) \\ &\quad + P(\widetilde{D} = 0 \mid H_0)\left(1 - \sum_{j=0}^{k-1}\binom{N}{j} P\left(u_i = 1 \mid \widetilde{D} = 0, H_0\right)^j P\left(u_i = 0 \mid \widetilde{D} = 0, H_0\right)^{N-j}\right) \\ &= \widetilde{P}_F\left(1 - \sum_{j=0}^{k-1}\binom{N}{j}(1 - p_0)^j p_0^{N-j}\right) + \left(1 - \widetilde{P}_F\right)\left(1 - \sum_{j=0}^{k-1}\binom{N}{j}(1 - p_1)^j p_1^{N-j}\right) . \end{aligned} \tag{12}$$

Similarly, the probability of detection can be written as[4]

$$P_D = \widetilde{P}_D\left(1-\sum_{j=0}^{k-1}\binom{N}{j}(1-p_0)^j p_0^{N-j}\right)+\left(1-\widetilde{P}_D\right)\left(1-\sum_{j=0}^{k-1}\binom{N}{j}(1-p_1)^j p_1^{N-j}\right). \tag{13}$$

2.3 FUSION NETWORK WITH FEEDBACK

Figure 3 shows a fusion network with feedback. On every time step, the N local processors communicate their decisions to the fusion center, and the fusion center then communicates the global decision back to the N local processors. The system operates as follows:

1. At time step t, the k^{th} processor makes a local decision u_k^t, based on the previous global decision D^{t-1}, the current observation, and all previous observations.
2. The fusion center receives all local decisions, and combines them to generate the global decision D^t.
3. The global decision is then transmitted to all local processors for use at time step $t + 1$.

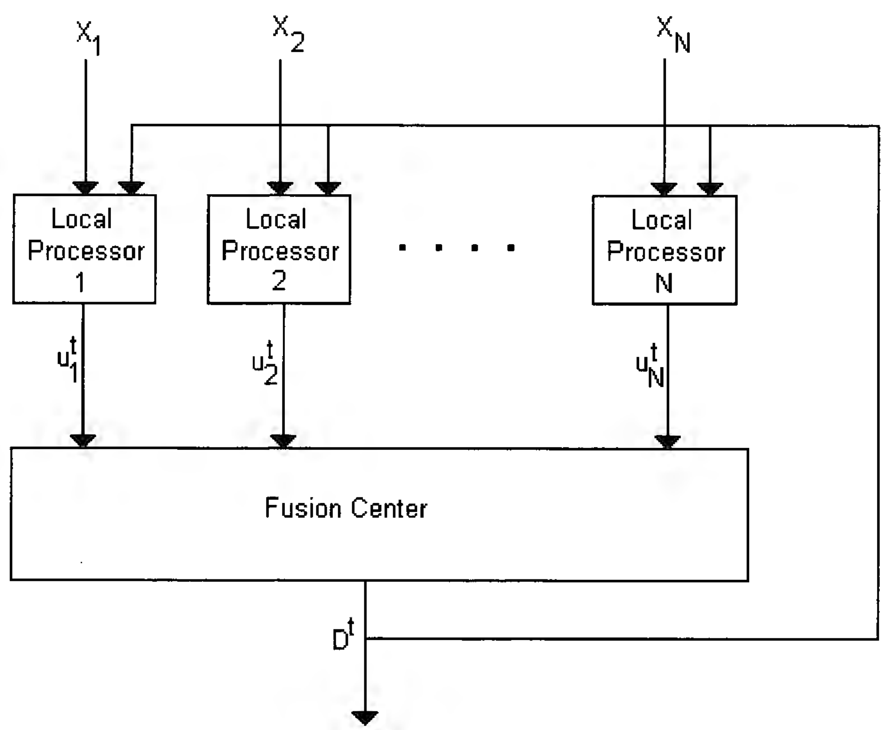

Figure 3. N-Processor Feedback Network

The N local processors generate a set of decisions $A_t = \left(u_1^t, u_2^t, \ldots, u_N^t\right)$, $t = 1,2,\ldots$ in the t^{th} processing interval. The binary decisions in A_t regard the hypotheses H_0 and H_1. The set A_t is combined at the fusion center to form D^t. From the discrete version of the Neyman-Pearson lemma, the most powerful combination rule at the fusion center is[7]

$$\phi_t(A_t) = \begin{cases} 1 & P(A_t \mid H_1) > \lambda_t P(A_t \mid H_0) \\ \gamma & = \\ 0 & < \end{cases}, \tag{14}$$

where $\gamma \in [0,1]$, and $\lambda_t \geq 0$ is the Lagrange multiplier.

At time step t, the probability of false alarm can be written as[7]

$$P_F^t = E\{\phi_t(A_t) \mid H_1\} = \begin{cases} P_F^{t-1} \sum_{A_t} \phi_t(A_t) P(A_t \mid D^{t-1} = 1, H_0) & \\ \quad + (1 - P_F^{t-1}) \sum_{A_t} \phi_t(A_t) P(A_t \mid D^{t-1} = 0, H_0) & t \geq 2 \\ \sum_{A_1} \phi_1(A_1) P(A_1 \mid H_0) & t = 1 \end{cases}. \tag{15}$$

Similarly, the probability of detection can be written as[7]

$$P_D^t = E\{\phi_t(A_t) \mid H_0\} = \begin{cases} P_D^{t-1} \sum_{A_t} \phi_t(A_t) P(A_t \mid D^{t-1} = 1, H_1) & \\ \quad + (1 - P_D^{t-1}) \sum_{A_t} \phi_t(A_t) P(A_t \mid D^{t-1} = 0, H_1) & t \geq 2 \\ \sum_{A_1} \phi_1(A_1) P(A_1 \mid H_1) & t = 1 \end{cases}. \tag{16}$$

An alternate topology for a fusion network with feedback uses parleying. This organization operates as follows:[8]

1. Each sensor receives an observation vector X_i.

2. Each sensor makes a tentative decision based on its own data, and transmits this decision to all the other sensors in the network.

3. Based on its own original data and the tentative decisions of all the other sensors, each sensor then "rethinks" and possibly revises its previous decision. This decision is then broadcast to all the other sensors in the network.

4. The parleying in step 3 continues until all sensors agree.

Although this topology has application in military analysis, it is not addressed in this document.

3. SERIAL NETWORK

Figure 4 shows an N-processor serial network. The first processor receives only an external observation vector X_1 and then outputs its decision u_1. The remaining processors receive an external observation vector X_i and a decision u_{i-1}, and then according to some fusion rule output decision u_i. Processor N outputs the decision of the network. There are two main reliability problems with a serial network.[2] The first, and most serious issue of the two, is communication failure. If a communication link fails anywhere in the chain, then all the processors before the failure are effectively removed from the system, resulting in serious performance degradation. The second problem is one of accumulated delays because each processor has to wait for input from the previous processors before generating a decision.

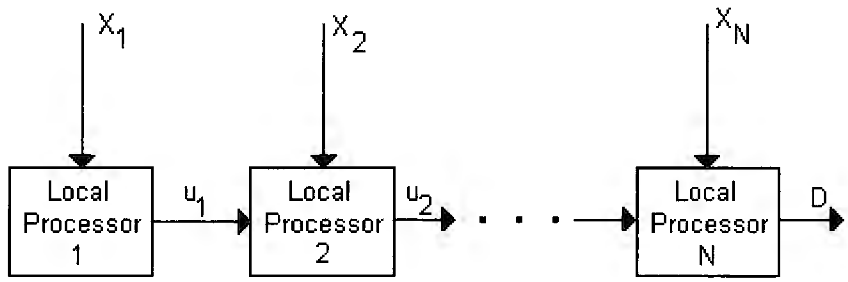

Figure 4. N-Processor Serial Network

For N serial processors, the probability of false alarm and probability of detection are computed recursively. Consider a serial network with two local processors. Processor one outputs decision u_1, and processor two outputs decision D. The network probability of false alarm is determined using the theorem of total probability[4]

$$\begin{aligned} P_F &= P(D=1 \mid H_0) \\ &= P(D=1 \mid u_1=1, H_0)P(u_1=1 \mid H_0) + P(D=1 \mid u_1=0, H_0)P(u_1=0 \mid H_0). \end{aligned} \tag{17}$$

Replacing H_0 with H_1 in equation (17) gives the probability of detection. The optimal thresholds for each processor are evaluated using the probabilities of false alarm and detection for the local processors. The thresholds for each processor are defined as α_i^j, the threshold used by processor i when it receives decision $u_{i-1} = j$. For the two-processor serial network, the complete set of threshold equations is [4]

$$\alpha_1 = \beta \frac{P_{f_2}(\alpha_2^1) - P_{f_2}(\alpha_2^0)}{P_{d_2}(\alpha_2^1) - P_{d_2}(\alpha_2^0)},$$

$$\alpha_2^0 = \beta \frac{1 - P_{f_1}(\alpha_1)}{1 - P_{d_1}(\alpha_1)},$$

$$\alpha_2^1 = \beta \frac{P_{f_1}(\alpha_1)}{P_{d_1}(\alpha_1)} \qquad . \tag{18}$$

Since $P(u_2 = 1 \mid u_1 = i, H_0) = P_{f_2}(\alpha_2^i)$, the global probability of false alarm is[4]

$$P_F = P_{f_2}(\alpha_2^1) P_{f_1}(\alpha_1) + P_{f_2}(\alpha_2^0)\left(1 - P_{f_1}(\alpha_1)\right). \tag{19a}$$

Similarly, the global probability of detection is[4]

$$P_D = P_{d_2}(\alpha_2^1) P_{d_1}(\alpha_1) + P_{d_2}(\alpha_2^0)\left(1 - P_{d_1}(\alpha_1)\right). \tag{19b}$$

Now consider an N-processor serial network. The serial network with j processors can be considered as a serial network of two processors. The first "processor" has the performance of the first j - 1 processors, while the second "processor" is the j^{th} processor of the network. The performance of local processor i is P_{d_i} and P_{f_i}, while the performance of a network with j processors is P_{D_j} and P_{F_j}. Recursive equation (20) provides the solution to the combined ROC curve for the network:[4]

$$\alpha_{j-1} = \beta_j \frac{P_{f_j}(\alpha_j^1) - P_{f_j}(\alpha_j^0)}{P_{d_j}(\alpha_j^1) - P_{d_j}(\alpha_j^0)},$$

$$\alpha_j^0 = \beta_j \frac{P_{F_{j-1}}(\alpha_{j-1})}{P_{D_{j-1}}(\alpha_{j-1})},$$

$$\alpha_j^1 = \beta_j \frac{1 - P_{F_{j-1}}(\alpha_{j-1})}{1 - P_{D_{j-1}}(\alpha_{j-1})},$$

$$\begin{aligned} P_{F_j} &= P_{F_{j-1}}(\alpha_{j-1}) P_{f_j}(\alpha_j^1) + \left(1 - P_{F_{j-1}}(\alpha_{j-1})\right) P_{f_j}(\alpha_j^0), \\ P_{D_j} &= P_{D_{j-1}}(\alpha_{j-1}) P_{d_j}(\alpha_j^1) + \left(1 - P_{D_{j-1}}(\alpha_{j-1})\right) P_{d_j}(\alpha_j^0), \end{aligned} \tag{20}$$

for $j = 2,\ldots,N$, where $P_{F_1} = P_{f_1}$, $P_{D_1} = P_{d_1}$, and β_j is the global threshold for a system of j processors computed as above. Note that these equations are coupled and, in general, are quite difficult to solve.

If the ROC curves for the local processors are symmetric, then the threshold equations are greatly simplified.[4] Symmetric local ROC curves imply that the global ROC curve is also symmetric. Thus, the global threshold $\beta = \beta_N = 1$ implies that $\alpha_j = 1$ for $j = 1,2,\ldots,N$-1. In addition, $P_{D_j} = 1 - P_{F_j}$ for $j = 1,2,\ldots,N$. As a result, the equations in (20) become[4]

$$
\begin{aligned}
&\alpha_{j\text{-}1} = 1, \\
&\alpha_j^0 = \frac{1 - P_{F_{j-1}}}{P_{F_{j-1}}}, \\
&\alpha_j^1 = \frac{P_{F_{j-1}}}{1 - P_{F_{j-1}}}, \\
&P_{F_j} = P_{F_{j-1}} P_{f_j}(\alpha_j^1) + \left(1 - P_{F_{j-1}}\right) P_{f_j}(\alpha_j^0), \\
&P_{D_j} = P_{D_{j-1}} P_{d_j}(\alpha_j^1) + \left(1 - P_{D_{j-1}}\right) P_{d_j}(\alpha_j^0),
\end{aligned}
\tag{21}
$$

for $j = 2,\ldots,N$, where $P_{F_1} = P_{f_1}$ and $P_{D_1} = P_{d_1}$. Equation (21) is then relatively easy to solve.

4. TREE NETWORK

A tree topology is more difficult to analyze than either the fusion or the serial network. Figure 5 shows a distributed detection network with a tree structure. Node N in figure 5 outputs D, the final decision of the network. A tree network can be represented by a directed acyclic graph with each node being a local processor. Edges represent one-way communication links between processors.

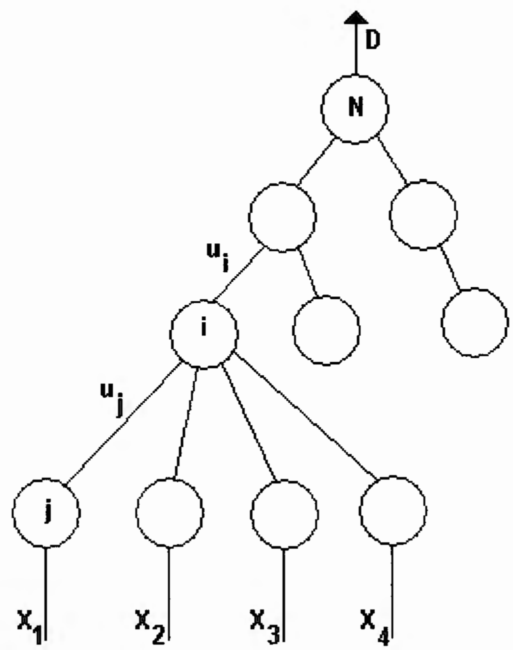

Figure 5. Distributed Detection Network in a Tree Structure (node N outputs final decision)

Each of the leaf nodes in a tree structure receives an observation vector X_i, makes a decision u_i based on that event, and transmits its decision to each of its immediate successor nodes. The internal root node receives decisions from all its predecessor nodes, combines the decisions according to some fusion rule, and transmits the decision to all its immediate successor nodes. For example, in figure 5, node j receives observation vector X_1, makes a decision u_j, and transfers this decision to node i. Node i receives four incoming decisions, combines them according to some fusion rule, and transmits decision u_i to its successor node.

$P_F(i) = P(u_i = 1|H_0)$ and $P_D(i) = P(u_i = 1|H_1)$ are, respectively, the probability of false alarm and the probability of detection of a subnetwork with node i as the root node. Using the theorem of total probability, if D is the final decision of the network, the probability of detection and the probability of false alarm can be written (independent of the optimality criterion) as[9,10]

$$P_D = P(D=1 \mid u_k = 0, H_1) + \left[P(D=1 \mid u_k = 1, H_1) - P(D=1 \mid u_k = 0, H_1)\right] P_D(k), \tag{22a}$$

and

$$P_F = P(D=1 \mid u_k = 0, H_0) + \left[P(D=1 \mid u_k = 1, H_0) - P(D=1 \mid u_k = 0, H_0)\right] P_F(k), \tag{22b}$$

for any node k where $k \neq N$.

5. BAYESIAN FORMULATION

Until this point, the network was assumed to follow a Neyman-Pearson optimality criterion. Now assume that the network follows a Bayesian minimum cost optimality criterion. The objective of the Bayesian criterion is to minimize the overall cost of the network decision. If $J(D, H_j)$ is the cost of the final processor deciding D when hypothesis H_j is true, optimality demands that the total expected cost $E[J(D, H_j)]$ be minimized. The functions that characterize the local processor decisions are chosen such that $E[J(D, H_j)]$ is a global minimum. This formulation could be difficult to implement in practice, since *a priori* the probabilities of each hypothesis and the costs associated with false alarms and detection are needed.

The only changes are for the network where the fusion center does not receive an observation vector, and the fusion center network with feedback. All the equations for the other networks remain the same. For the fusion center network where the fusion center does not receive an external observation vector (see section 2.1), the only change is to β,[11] or

$$\beta = \frac{P(H_0)[J(1,H_0) - J(0,H_0)]}{P(H_1)[J(0,H_1) - J(1,H_1)]}. \tag{23}$$

In section 2.3, the global probability of false alarm and the global probability of detection change. The system probability of error is[12]

$$P_e^t = P_F^t P(H_0) + P_D^t P(H_1). \tag{24}$$

Expanding P_F^t in terms of D^{t-1} [12]

$$\begin{aligned} P_F^t = {} & P(D^t = 1 \mid H_0) + P(D^t = 1 \mid D^{t-1} = 1, H_0) P(D^{t-1} = 1 \mid H_0) \\ & + P(D^t = 1 \mid D^{t-1} = 0, H_0) P(D^{t-1} = 0 \mid H_0) \end{aligned} \quad . \tag{25}$$

Replacing $P(D^{t-1} = 0 \mid H_0)$ with $1 - P(D^{t-1} = 1 \mid H_0)$ and rearranging terms yields[12]

$$\begin{aligned} P_F^t = {} & P(D^{t-1} = 1 \mid H_0)[P(D^t = 1 \mid D^{t-1} = 1, H_0) - P(D^t = 1 \mid D^{t-1} = 0, H_0)] \\ & + P(D^t = 1 \mid D^{t-1} = 0, H_0) \end{aligned} \quad . \tag{26}$$

Thus, the global probability of false alarm on time step t can be written as[12]

$$\begin{aligned} P_F^t & = P(D^t = 1 \mid H_0) \\ & = P_F^{t-1}[P_F^t(D^{t-1} = 1) - P_F^t(D^{t-1} = 0)] + P_F^t(D^{t-1} = 0) \end{aligned} \cdot \tag{27}$$

Similarly, the global probability of detection on time step t is[12]

$$P_D^t = P\left(u_0^t = 0 \mid H_0\right)$$
$$= 1 - P_D^{t-1}\left[P_D^t\left(u_0^{t-1} = 0\right) - P_D^t\left(u_0^{t-1} = 1\right)\right] + P_D^t\left(u_0^{t-1} = 1\right) \cdot \tag{28}$$

6. SUMMARY

This document surveyed work on how to combine multiple ROC curves into a single network ROC curve for three network topologies. In warfare analysis, obtaining a single descriptor of a system is critical. As the focus on networking and information technology in warfare analysis intensifies, the ability to describe a system quickly and easily will be imperative. A network ROC curve is a rapid and simple way to explain the performance of a system. While many of the formulas presented here are difficult to solve, through parametric analysis they can provide insight into trends in various warfare areas where ROC curves arise, including detection/classification, command and control, and decision-making.

APPENDIX A
APPROXIMATION OF INCOMPLETE BETA FUNCTION

When $(a + b - 1)(1 - x) \geq 0.8$, the incomplete beta function can be approximated by the complement of the cumulative Gaussian distribution function.[5] If $a + b > 6$, then[5]

$$I_x(a,b) = Q(y) + \varepsilon = \frac{1}{\sqrt{2\pi}} \int_y^{\infty} e^{\frac{-t^2}{2}} dt + \varepsilon, \tag{A-1}$$

where $|\varepsilon| < 5 \times 10^{-3}$ and

$$y = \frac{3\left[w_2 (1 - \frac{1}{9a}) - w_1 (1 - \frac{1}{9b}) \right]}{\sqrt{\frac{w_1^2}{b} + \frac{w_2^2}{a}}}, \tag{A-2}$$

with $w_1 = (bx)^{1/3}$ and $w_2 = (a(1 - x))^{1/3}$. In turn, y can be written as

$$y = \frac{(1-x)^{1/3}(9a-1)b^{2/3} - x^{1/3}(9b-1)a^{2/3}}{3a^{1/2}b^{1/2}\sqrt{x^{2/3}a^{1/3} + (1-x)^{2/3}b^{1/3}}}. \tag{A-3}$$

REFERENCES

1. Z. Chair and P. K. Varshney, "Optimal Data Fusion in Multiple Processor Detection Systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-22, no. 1, January 1986, pp. 98–101.

2. R. Viswanathan and P. K. Varshney, "Distributed Detection with Multiple Processors: Part I – Fundamentals," *Proceedings of the IEEE*, vol. 85, no. 1, January 1997, pp. 54–63.

3. Harry L. Van Trees, *Detection, Estimation, and Modulation Theory, Part I,* John Wiley & Sons, New York, 1968.

4. Amy Reibman, "Performance and Fault Tolerance of Distributed Detection Networks," Ph.D. dissertation, Department of Electrical Engineering, Duke University, Durham, NC, 1987.

5. Milton Abromowitz and Irene Stegun, eds. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables,* National Bureau of Standards, Washington, DC, 1970.

6. Amy Reibman and Loren Nolte, "Performance of Redundant and Distributed Detection Systems with Processor Faults," *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, 1998, pp. 2805–2808.

7. R. Srinivasan, "Distributed Detection with Decision Feedback," *IEE Proceedings on Radar and Signal Processing*, vol. 137, pt. F, no. 6, December 1990, pp. 427–432.

8. Peter F. Swaszek and Peter Willett, "Parley as an Approach to Distributed Detection," *IEEE Transactions on Aerospace and Electronic Systems,* vol. 31, no. 1, January 1995, pp. 447–457.

9. Z. Tang, K. Pattipati, and D. Kleinman, "Optimization of Detection Networks: Part II – Tree Structures," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 23, no. 1, January 1993, pp. 211–221.

10. Ming Xiang and Chongzhao Han, "Optimization of Distributed Detection Networks with Tree Structures," *Proceedings of the 5th International Conference on Information Fusion 2002*, vol. 1, July 2002, Annapolis, MD, pp. 164–169.

11. Jason Papastavrou and Michael Athans, "On Optimal Distributed Decision Architectures in a Hypothesis Testing Environment," *IEEE Transactions on Automatic Control*, vol. 37, no. 8, August 1992, pp. 1154–1169.

12. S. Alhakeem and P. K. Varshney, "Decentralized Bayesian Detection with Feedback," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 26, no. 4, July 1996, pp. 503–513.

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